THE SMALL SIGNAL AC IMPEDANCE OF A SHORT p–n JUNCTION DIODE

G. GARCIA-BELMONTE1, J. BISQUERT1 and V. CASELLES2
1Departament de Ciències Experimentals, Universitat Jaume I, 12080 Castelló, Spain
2Departament de Matemàtiques i Informàtica, UIB, 07071 Palma (Balears), Spain

(Received 10 July 1997; in revised form 28 November 1997)

1. INTRODUCTION

The small signal AC impedance of a uniformly doped p–n junction is[1]

\[ Z_{pn}(\omega) = R_0 \frac{\tanh\left(\frac{\tau_n/\tau_p + j\omega \tau_n}{1 + j\omega \tau_p}\right)}{1 + j\omega \tau_n} \] (1)

when a direct voltage is applied under low injection conditions. In Equation (1) \( \omega \) is the angular frequency \( \omega = 2\pi f \), where \( f \) is the frequency, \( R_0 = (dV/dI)^{-1} \), \( \tau_p \) is the lifetime of the hole minority carriers, \( w \) is the length of the lower doped \( n \) bulk region, \( L_p \) is the minority carrier diffusion length for holes, with \( \tau_p = L_p^2/D_p \), where \( D_p \) is the diffusion coefficient, and finally \( \tau_n = w^2/D_n \) is a characteristic transit time.

When the lower doped \( n \) region of the diode is narrow \( (w < L_p) \) the impedance in Equation (1) can be approximated in the whole frequency range by the function

\[ Z_{ pnw}(\omega) = R_0 \frac{\tanh\left(\frac{\sqrt{j\omega \tau_n}}{\sqrt{j\omega \tau_p}}\right)}{\sqrt{j\omega \tau_n}} \] (2)

which was discussed some time ago by Lindmayer and Wrigley[2]. This impedance function is often used in the treatment of diffusion phenomena along thin spatial regions in electrochemical systems[3,4].

The use of \( Z_{pnw} \) instead of \( Z_{pn} \) greatly simplifies the equivalent circuit approach to characterise a p–n junction, as we have shown in a recent paper[5]. The measurement of the impedance as a function of frequency is straightforward, and a fit to a model circuit in which \( Z_{pnw} \) accounts for diffusion of minority carriers provides an accurate determination of the parameter \( \tau_n \).

Here we address some issues that are important for the application of Equation (2) in impedance spectroscopy studies of p–n junctions. We present a rigorous proof of the validity of the approximation, and we discuss the small signal ac response of the junction in terms of a finite transmission line length. Finally, we determine the error involved when using \( Z_{pnw} \) in place of \( Z_{pn} \).

2. FORMAL STUDY OF THE APPROXIMATION

To derive Equation (2) from Equation (1) in a simple way we state the short diode condition as \( \beta \ll 1 \), where \( \beta \) is

\[ \beta = \frac{\tau_n}{\tau_p} = \left( \frac{w}{L_p} \right)^2 \] (3)

In Equation (1) we expand the term \( \tanh(\sqrt{\beta}) \) using the formula

\[ \tanh(z) = z - \frac{z^3}{3} + \cdots \] (4)

and we get

\[ Z_{pn}(\omega) \approx R_0 \frac{\tanh\left(\frac{\tau_n/\tau_p + j\omega \tau_n}{1 + j\omega \tau_p}\right)}{\tau_n/\tau_p + j\omega \tau_n} \] (5)

If the small term \( \tau_n/\tau_p \) is neglected, then Equation (5) turns into \( Z_{pnw} \).

A rigorous proof of the validity of the \( w \)-approximation goes as follows. Let us define the dimensionless functions \( \Theta_{pn} = Z_{pn}/R_0 \) and \( \Theta_{pnw} = Z_{pnw}/R_0 \) and consider the inequality

\[ |\Theta_{pn} - \Theta_{pnw}| \leq |\Theta_{pn} - f(j\Omega + \beta)| + |f(j\Omega + \beta) - f(j\Omega)| \] (6)

where \( \Omega = \omega \tau_n \) and \( f \) is the complex function

\[ f(z) = \frac{\tanh(\sqrt{z})}{\sqrt{z}} \] (7)

Taking Equation (5) into account, we get from Equation (6)

\[ |\Theta_{pn} - \Theta_{pnw}| \leq \frac{\beta}{2} + |f(j\Omega + \beta) - f(j\Omega)| \] (8)

A theorem of complex variable calculus ensures us that

\[ |f(j(\Omega + \beta) - f(j\Omega)| \leq |f'(\zeta)|\beta \] (9)

where \( \zeta \) is a point in the line \( \Lambda = [j\Omega, j(\Omega + \beta)] \). It is easily checked that the derivative
\[ f'(z) = \frac{1}{2z} \left( \frac{1}{\cosh(\sqrt{z})} \frac{\tanh(\sqrt{z})}{\sqrt{z}} \right) \quad (10) \]

takes its maximum value in the line \( \lambda = \beta \Omega \).
Furthermore, \( |f(\beta \Omega)| \) is a decreasing function of \( \Omega \), and therefore the quantity
\[
\lim_{\Omega \to 0} |f'(\beta \Omega)| = \frac{1}{3}
\]
is an upper bound to the derivative. Thus, we have shown that
\[
|\Theta_p - \Theta_{p_{nw}}| \leq \frac{2\beta}{3} \quad (11)
\]
For instance, when \( \frac{w}{L_p} = 0.1 \), the bound is 0.007.

### 3. STRUCTURE OF THE IMPEDANCE FUNCTIONS

As is well known, recombination phenomena in the narrow, lower doped \( n \) region of the diode are negligible compared to diffusion phenomena. It is not surprising that the impedance \( Z_{p_{nw}} \) is the exact impedance function of 1D diffusion in a finite length region in which the diffusion species is drained out at the boundary. The impedance \( Z_{p_{nw}} \) is sometimes represented as an RC parallel combination where both the resistance and capacitance change with frequency[2]. Nevertheless, it is more useful to consider the distributed circuit represented in Fig. 1, owing to the fact that in terms of the impedance, non-blocked 1D diffusion in a finite line is analogous to a finite length transmission line terminated in a short-circuit represented in Fig. 1. The effective signal penetration length in the line of Fig. 1 is of the order of \( \lambda = 1/\sqrt{\tau_c \epsilon} \), and in consequence the impedance has two distinct frequency regimes. For high enough frequencies so that the penetration length is shorter than the length of the line, the impedance is effectively the same as in diffusion in a semi-infinite zone, whereas at low frequencies the distributed circuit behaves as a \( RC \) parallel association. These trends are indeed encountered in the typical shape of the function \( Z_{p_{nw}} \), which is presented in Fig. 2. The two frequency regimes are separated by the characteristic frequency \( \omega_n = 2\pi \tau_w^{-1} \). At high frequencies (HF) \( \omega >> \omega_n \),

\[
Z_{p_{nw}}(\omega) \approx R_0 \left[ 1 + \frac{1}{3} \frac{\omega}{\tau_p} \left( \frac{1 + \frac{\tau_w}{\tau_p}}{3} \right) \right] \quad (14)
\]

while at HF, Equation (1) in the admittance form turns into

\[
Y_{p}(\omega) \approx \frac{1}{\sqrt{\omega \tau_p \tanh(\tau_w/\tau_p)^{1/2}}} \quad (15)
\]

### 4. ERROR INVOLVED IN THE USE OF THE APPROXIMATION

The previous results allow to establish accurately the amount of error committed when using \( Z_{p_{nw}} \) instead of \( Z_{p_{nw}} \). We will consider the relative error
\[
\Delta_p(\omega, \tau_p, \tau_w) = \frac{|Z_{p_{nw}} - Z_{p_{nw}}|}{|Z_{p_{nw}}|} \quad (16)
\]
as one usually employs the modulus of the impedance as a weight function in the fit of impedance data[6]. In all cases where \( \beta << 1 \), the relative error \( \Delta_p \) presents the following characteristics: it is negligible at LF, peaks at intermediate frequencies \( \omega \approx \omega_n \), and attains a constant value at HF. This is seen in the results of the numerical calculation of \( \Delta_p \) for \( \beta = 0.05 \) and \( \beta = 0.1 \) presented in Fig. 3. To explain the typical shape of \( \Delta_p \), it can be
observed that at LF the real parts of Equations (13) and (14) are the same, and that the difference between the respective imaginary parts gets smaller as $\omega$ decreases, therefore for small frequency values $\Delta_u$ becomes practically zero. At HF, a comparison of Equations (12) and (15) shows that in this frequency regime the error is independent of frequency. Expanding the tanh term in Equation (15) one can compute the relative error at HF, yielding

$$\Delta_u(HF) = \frac{\beta}{3}$$

This general result is fulfilled by the numerical calculations presented in Fig. 3. Equation (17) is valid only for a short diode, as the numerical calculations of $\Delta_u$ for $\beta = 1$ and $\beta = 10$ presented in Fig. 4 show. Finally, it should be remarked that the approximate expression in Equation (13), which is often employed as an approximation to $Z_{pn}$ for a short diode[7], is useless at HF.

Acknowledgements—The authors at Universitat Jaume I appreciate the support of A. Segura. This work was partially supported by the Fundació Caixa-Castelló. An anonymous reviewer is acknowledged for bringing Ref.[2] to our attention.

REFERENCES